UDC 368.519.2

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QUANTITATIVE METHODS ESTIMATION OF THE COMPETITIVENESS OF INSURANCE COMPANIES IN THE CONTEXT OF SUSTAINABLE DEVELOPMENT

Abstract. The presence of a large number of insurance companies in the Ukrainian insurance market requires the use of quantitative methods to assess their competitiveness and attractiveness for consumers of insurance services. In order to ensure the level of competitiveness of an insurance company that would meet the needs of policyholders in comparison with other companies, it is necessary to orient the activity on the line «sustainable development». The use of quantitative methods in assessing the competitiveness of insurance companies is due to the fact that their application is based on well-defined hypotheses in the presence of specific quantitative indicators and obtaining results with a high degree of accuracy.

The priority objective of an insurance company is to obtain the greatest number of competitive advantages that serve as the basis for ensuring a constant level of efficiency, profitability and profitability of their activities. Solving this problem is possible with the use of an economic and mathematical model aimed at the iterative formation of the structure of the insurance portfolio. There are different methods of optimizing the activities of companies that allow you to increase the amount of profit and increase profitability. One of the methods that an insurance company can use to improve its efficiency is optimization of the insurance portfolio, which includes determining the rational combination of the proportion of individual types of services in the portfolio, analysis of solvent demand for certain types of services, taking into account the restrictions and needs of the market, the choice of rational combination prices and sales volumes, taking into account market demand. By developing a model for quantifying the performance of an insurance company to determine its level of competitiveness, quantitative significance was obtained which allowed ranking the insurance companies according to this criterion. The obtained results allowed determining the competitive advantages of insurance companies and determining their competitiveness in the market.

Keywords: quantitative methods, sustainability, competitiveness, insurance companies, insurance market, sustainable development.

JEL Classification C53, G22

Formulas: 26; fig.: 5; tabl.: 5; bibl.: 13.

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ISSN 2306-4994 (print); ISSN 2310-8770 (online)

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КІЛЬКІСНІ МЕТОДИ ОЦІНКИ КОНКУРЕНТОСПРОМОЖНОСТІ СТРАХОВИХ КОМПАНІЙ У КОНТЕКСТІ СТАЛОГО РОЗВИТКУ

Анотація. Наявність на українському страховому ринку великої кількості страхових компаній вимагає використання кількісних методів оцінки їхньої конкурентоспроможності та привабливості для споживачів страхових послуг. Для забезпечення рівня конкурентоспроможності страхової компанії, який би відповідав потребам страхувальників порівняно з іншими компаніями, потрібно орієнтувати діяльність на сталий розвиток. Використання кількісних методів оцінки конкурентоспроможності страхових компаній пояснюється тим, що їх застосування базується на чітко визначених гіпотезах за наявності конкретних кількісних показників та отримання результатів із високим ступенем точності.

Пріоритетним завданням страхової компанії є отримання найбільшої кількості конкурентних переваг, які служать основою для забезпечення постійного рівня ефективності, рентабельності та прибутковості їхньої діяльності. Розв’язати цю проблему можливо з використанням економіко-математичної моделі, спрямованої на ітераційне формування структури страхового портфеля. Існують різні методи оптимізації діяльності компаній, які дозволяють збільшити розмір прибутку і підвищити прибутковість. Одним із методів, який може використовувати страхова компанія для підвищення своєї ефективності, є оптимізація страхового портфеля, що включає визначення раціонального поєднання частки окремих видів послуг у портфелі, аналіз платоспроможного попиту на певні види послуг, прийняття, ураховуючи обмеження та потреби ринку, вибір раціональних комбінованих цін і обсягів продажу з урахуванням попиту на ринку. Розробляючи модель кількісної оцінки показників діяльності страхової компанії для визначення рівня її конкурентоспроможності, було отримано кількісне значення, що дозволило класифікувати страхові компанії за цим критерієм. Отримані результати дозволили визначити конкурентні переваги страхових компаній та їхню конкурентоспроможність на ринку.

Ключові слова: кількісні методи, стійкість, конкурентоспроможність, страхові компанії, страховий ринок, сталий розвиток.

Формул: 26; рис.: 5; табл.: 5; бібл.: 13.

Introduction. The insurance market as a leading link in the financial system of the country has specific functions and features that influence the formation of the results of its activities and provide economic and social benefits. During the last decennary, the Ukrainian insurance market has undergone significant changes in the qualitative and quantitative composition of insurance companies that provide the process of providing insurance services to individuals and legal entities. In accordance with the marked structural fluctuations, a serious problem arises in ensuring an adequate level of competition between insurance companies and the drift of insurers that have formed a customer base between them for this time period. The use of different tools to increase the competitive advantages of an insurance company in the modern insurance market will ensure not only its survival but also sustainable development.

The need to ensure the competitiveness of an insurance company through the sublimation of elements of competitive advantages such as: compliance with the structure of the use of financial resources for the implementation of insurance services within the competitive environment; observance of geographical diversification in all regions and regions of the country; spectrum of available insurance products; the use of price and non-price competition; providing quality insurance compensation service. It should be noted the lack of disclosure of scientific and methodological approaches to quantitative assessment of the competitiveness of insurance companies.

Literature Review. The question of the quantitative assessment of the competitiveness of insurance companies, the feasibility of using methods of mathematical modeling in this area are devoted to the work of many American, European and Ukrainian scientists, in particular G. M. Azarenkova [1], O. V. Kozmenko [11], A. Kozhukhivska, A. V. Monakhova, E. V. Shikina, M. Morawetz, H. Schmeiser, T. Störmer, J. Wagner, C. Biener, M. Eling, B. Berliner, N. Bühlmann, Mandić Ksenija, Delibašić Boris, Knežević Snežana, Benković Sladjana, Anastasios Magoutas, PanosChountalas, Zhongyi Yuan, Anna Szymańska and other authors.

The popularity of using quantitative methods for assessing the competitiveness of insurance companies is due to the fact that their application is based on well-defined hypotheses in the presence of specific quantitative indicators and obtaining results with a high degree of accuracy. The basic vector for developing a quantitative model for assessing the competitiveness of insurance companies is the popularization of the philosophy of sustainable development. The hypothesis of this research is the following: if, based on the results of the calculations, the insurance company has a sufficient number of competitive advantages, then it is competitive in compliance with the quantitative parameters of the formation of the insurance portfolio.

The formulated hypothesis in quantitative terms will allow for the combination of effective performance indicators of the insurance company to assess its competitive position in the insurance market.

One of the main tasks of an insurance company is to get the most competitive advantages that serve as the basis for ensuring a constant level of efficiency, profitability and profitability of their activities. The competitiveness of the insurance company is determined by many as internal factors, and external factors of the competitive environment, which are interrelated. Optimization of the internal factors of the insurance company in connection with the boundary factors of the external environment is important for the assessment and enhancement of the competitiveness.

First, we focus on the main characteristics of the effectiveness of the operation of any company, including insurance, that is, on the indicator of profitability of activity. There are different methods of optimizing the activities of companies that allow you to increase the amount of profit and increase profitability. One of the methods that an insurance company can use to improve its effectiveness is the optimization of the range of services (optimization of the insurance portfolio), which includes determining the rational combination of the proportion of individual types of services in the portfolio, analysis of solvent demand for certain types of services, taking into account constraints and needs the market, the choice of a rational combination of prices and sales volume in the light of market demand.

The research of optimization processes to increase the effectiveness of management activities in the insurance company is quite risky. That is why the construction of an economic-mathematical model, which serves as a simplified reflection of the financial flows of a truly active insurance company, and characterizes only the most important aspects for the researcher, makes it possible to determine the essential properties and behavior in any probable situations.

Methodology and research methods. Determine the exercise of economics-mathematical modeling of optimizing the internal factors of the insurance company's activity to assess the level of competitiveness in relation to the boundary conditions of the environment as a problem of linear programming. This allows you to significantly simplify the calculations, without neglecting the most important characteristics of this study.

Parameters c_l, z_l, v_l, w_l, are quantitative characteristics of the activities of the insurance company. Some of these parameters, for example c_l, may vary depending on the change in the

ISSN 2306-4994 (print); ISSN 2310-8770 (online)

tactics of the company and, accordingly, obtaining new competitive advantages or the loss of existing ones.Another part of the parameters z_l, v_l, w_l is conditionally constant because they remain constant within the short and medium term of the insurance company's operation in the conditions of the existing competitive environment. Changing the strategy can affect the significant changes in all system parameters.

Incoming, and in this case, the sought-after, variable models are the shares of insurance services in the insurance portfolio x_l of the company, which are managed variables, since they are determined not only by the external environment, but also to a large extent dependent on managerial decisions.An important and determining part of the construction of any economic-mathematical model is the target function. Within the optimization of the internal factors of the insurance company's activity, in conjunction with the boundary conditions of the environment, the objective function is to maximize the level of competitiveness, which is determined by the number of competitive advantages caused by the provision of certain insurance services:

$$F = \sum_l c_l x_l \to \max. \quad (1)$$

Opportunities to choose the fractions of insurance services in the insurance portfolio of the company are always limited to the limit conditions, such as insurance reserves (reflect the degree of protection of the insurance company in case of occurrence of insurance cases, that is, the degree of protection from negative influences), solvent demand on the insurance market in value terms (reflects the maximum possible increase in the cost of providing insurance services), insurance tariffs (reflect the degree of adaptation of the insurance company to external conditions of functioning within the existing the competitive environment) and so on.

So, let's consider the limitations of the model, namely:

– the sum of all the fractions of the insurance services in the insurance portfolio of the company is equal to one, that is, the optimal insurance portfolio consists of a certain number of insurance services that form the given portfolio:

$$\sum_l x_l = 1; \quad (2)$$

– the cost of covering losses in case of occurrence of a certain insurance event is carried out within the limits of insurance reserves of the company (SR),they must not exceed the reserve data, which provides profitable (or at least not loss-making) operation in the insurance market:

$$\sum_l z_l x_l \le SR; \quad (3)$$

– the cost of providing insurance services should not exceed the cost of measuring the effective demand (PP):

$$\sum_l v_l x_l \le PP; \quad (4)$$

– the cost of conducting business for the provision of insurance services must be provided by the insurance company established by insurance rates (ST):

$$\sum_l w_l x_l \le ST; \quad (5)$$

– the integral part of insurance services in the insurance portfolio of the company as an economic variable:

$$x_l \ge 0. \quad (6)$$

When constructing an economic-mathematical model for optimizing the internal factors of the insurance company's activity in relation to the boundary conditions of the environment, the main principles were taken into account, namely:

– adequacy of the constructed model to the real financial flows of the insurance company, that is, the processes of receipt and disposal of financial resources of the company for a certain period of time;

– the model takes into account significant (in the framework of this research) parameters and neglected insignificant, secondary;

– clarity of the constructed model for its users, that is, persons who make managerial decisions regarding the operation of an insurance company;

– content of the set of wanted variables — the shares of insurance services in the insurance portfolio of the company x_l.

The economic-mathematical model as a exercise of linear programming optimizing the internal reserves of an insurance company to assess the level of competitiveness in relation to the boundary conditions of the environment takes the following form:

$$F = \sum_l c_l x_l \to \max; \tag{7}$$

$$\begin{gathered} \sum_l x_l = 1 \\ \sum_l z_l x_l \le SR \\ \sum_l v_l x_l \le PP, \\ \sum_l w_l x_l \le ST \\ x_l \ge 0 \end{gathered} \tag{8}$$

F – the level of competitiveness of an insurance company;

x_l – the fractions l insurance service in the insurance portfolio;

c_l – competitive advantages that make it possible to use l insurance service;

z_l – the cost of covering losses that arise in the event of the occurrence of insurance incidents provided for by the provision l insurance service;

SR – insurance reserves of the company;

v_l – cost of provision l insurance service;

PP – solvent demand on the insurance market in terms of value;

w_l – transaction costs l insurance service;

ST – established by the insurance company rate l insurance service.

The solution of the problem of linear programming involves finding the optimal strategy of the given economic system by checking all possible combinations of insurance services in the insurance portfolio of the company $x_1, x_2, ..., x_l, ..., x_m$, which satisfy the conditions (8). For some part of the admissible plan, conditions (8) are executed as equality, therefore, characterize the boundary points of the area of solution of the problem. Each of these admissible plans is called the reference plan. Accordingly, the optimal plan will be acceptable $x_1*, x_2*, ..., x_l*, ..., x_m*$, at which the insurance company achieves the maximum possible level of competitiveness in the functioning within the framework of the competitive environment in the insurance market.

The real economic problems of linear programming in general and the task of optimizing the internal factors of an insurance company’s activity to assess the level of competitiveness in conjunction with the boundary conditions of the environment in particular have a fairly large dimension, and the process of selecting all possible combinations of solutions is difficult and time consuming, which leads to the loss of adequacy of the optimal plan found.

Results. The most common method of solving linear programming problems, which provides the ability to obtain accurate results over a fairly short period of time, is a simplex method. The essence of this method is to carry out an iterative procedure, which leads to a search of such a reference plan at each step, in which the value of the target function is at least worse found in the previous step, that is, during the solution of the problem, gradually the value of the functional changes in the direction of increase (for tasks at maximum).

The algorithm for solving the linear programming problem using the simplex method involves the implementation of five steps:

ISSN 2306-4994 (print); ISSN 2310-8770 (online)

1. Formation of the reference plan of the problem of linear programming by bringing it to the canonical form and allocation of a unit matrix, which will characterize the basic variables — the initial reference plan, and the corresponding value of the objective function.

2. Construction of a simplex table that reflects the formalization of the initial reference plan of the linear programming problem.

3. Checking the reference plan for optimality with the help of ratings that must be integral to the solution of the linear programming problem, which involves maximizing the target function. If all estimates satisfy the optimality condition, then the defined reference plan is the optimal task plan. If at least one of the estimates does not satisfy the optimality conditions, then they move to a new reference plan or establish that the optimal task plan does not exist.

4. The transition to the new reference plan of the problem is carried out by changing the basis, that is, the basis introduces a variable that improves the value of the target function, and displays the variable that it worsens, and calculate the elements of the new simplex table.

5. Repeat actions starting from 3 points, that is, checking the received new support plan for optimality, and in case of its non-optimality — search for a new reference plan that greatly improves the target function of the linear programming problem.

The first stage of the application of the simplex method to optimize the insurance portfolio of the company by determining the fraction l insurance service is the reduction of the constructed linear programming problem to the canonical form, which involves the transformation of all non-strict constraints by the strict introduction of additional fictitious variables $x_{n+1}, x_{n+2}, x_{n+3}$. In this case, the target function fictitious variables are introduced with zero coefficients. With these transformations, the economics-mathematical model takes on the following form:

$$\max F = c_1x_1 + c_2x_2 + ... + c_lx_l + ... + c_nx_n + 0x_{n+1} + 0x_{n+2} + 0x_{n+3}; \qquad (9)$$

$$\begin{cases} z_1x_1 + z_2x_2 + ... + z_lx_l + ... + z_nx_n + x_{n+1} = SR \\ v_1x_1 + v_2x_2 + ... + v_lx_l + ... + v_nx_n + x_{n+2} = PP \\ w_1x_1 + w_2x_2 + ... + w_lx_l + ... + w_nx_n + x_{n+3} = ST \\ x_1 + x_2 + ... + x_l + ... + x_n = 1 \end{cases}. \qquad (10)$$

$$x_l \geq 0.$$

According to the simplex method, it is necessary to identify the basic variables that determine the optimal plan and improve the value of the target function. The system of equations (10) contains only 3 single vectors, so it does not have a single matrix. Therefore, we use the method of an artificial basis, according to which one unit matrix can be obtained, if the fourth equation $x_1 + x_2 + ... + x_l + ... + x_n = 1$ in the limitations of our task add one artificial variable $x_{n+4} \geq 0$:

$$\begin{cases} x_{n+1} + z_1x_1 + z_2x_2 + ... + z_lx_l + ... + z_nx_n = SR \\ x_{n+2} + v_1x_1 + v_2x_2 + ... + v_lx_l + ... + v_nx_n = PP \\ x_{n+3} + w_1x_1 + w_2x_2 + ... + w_lx_l + ... + w_nx_n = ST \\ x_{n+4} + x_1 + x_2 + ... + x_l + ... + x_n = 1 \end{cases}. \qquad (11)$$

The introduction of an artificial variable affects the appearance of the target function - to exclude from the basis of an artificial variable, it is necessary to enter it into a target function with a negative coefficient M. The magnitude M is a fairly large number. That is, the target function will on the following form:

$$\max F = c_1x_1 + c_2x_2 + ... + c_lx_l + ... + c_nx_n + 0x_{n+1} + 0x_{n+2} + 0x_{n+3} - Mx_{n+4}. \qquad (12)$$

The constraint system (11) in the vector form takes on the following form:

$$x_1A_1 + x_2A_2 + x_3A_3 + ... + x_nA_n + x_{n+1}A_{n+1} + x_{n+2}A_{n+2} + x_{n+3}A_{n+3} + x_{n+4}A_{n+4} = A_0, \qquad (13)$$

$A_1 = \begin{pmatrix} z_1 \\ v_1 \\ w_1 \\ 1 \end{pmatrix}$, $A_2 = \begin{pmatrix} z_2 \\ v_2 \\ w_2 \\ 1 \end{pmatrix}$, …, $A_n = \begin{pmatrix} z_n \\ v_n \\ w_n \\ 1 \end{pmatrix}$ – vectors of a 4-dimensional space, forming a matrix of coefficients before the variables of the system of limitations of the linear programming problem;

$A_{n+1} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $A_{n+2} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$, $A_{n+3} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$, $A_{n+4} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ – linearly independent single vectors of a 4-dimensional space forming a unit matrix and forming the basis of this space;

$A_0 = \begin{pmatrix} SR \\ PP \\ SP \\ 1 \end{pmatrix}$ — a vector of 4-dimensional space, forming a matrix of free members of the system of limitations of the linear programming problem.

From formula (13) the basis variables will be x_{n+1}, x_{n+2}, x_{n+3}, x_{n+4}, and the other variables are free. Equall all free variables to zero, that is $x_1 = 0, x_2 = 0, ..., x_n = 0$. The first support plan takes the following form, which is one of the solutions to the constraints system (11):

$$X_0 = (x_1 = 0, x_2 = 0, ..., x_n = 0, x_{n+1} = SR, x_{n+2} = PP, x_{n+3} = SP, x_{n+4} = 1). \quad (14)$$

The next stage of the solution of the problem of optimizing the structure of the insurance portfolio is the construction.

This row of a simplex table will be called the estimated value in the future.

Estimates of the optimal reference plan for each variable:

– for the variable x_1: $0z_1 + 0v_1 + 0w_1 - M \cdot 1 - c_1 = -M - c_1$;

– for the variable x_2: $0z_2 + 0v_2 + 0w_2 - M \cdot 1 - c_2 = -M - c_2$;

– for the variable x_l: $0z_l + 0v_l + 0w_l - M \cdot 1 - c_l = -M - c_l$;

– for the variable x_n: $0z_n + 0v_n + 0w_n - M \cdot 1 - c_n = -M - c_n$;

– for the variable x_{n+1}: $0 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 - M \cdot 0 - 0 = 0$;

– for the variable x_{n+2}: $0 \cdot 0 + 0 \cdot 1 + 0 \cdot 0 - M \cdot 0 - 0 = 0$;

– for the variable x_{n+3}: $0 \cdot 0 + 0 \cdot 0 + 0 \cdot 1 - M \cdot 0 - 0 = 0$;

– for the variable x_{n+4}: $0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 - M \cdot 1 - (-M) = 0$.

The next step in finding the optimal plan for the problem of linear programming is to check the received reference plan, the formalization of which is given in table 2, for optimality. Thus, the condition of optimality of the task for maximizing the target function is formulated as follows:

- if for a certain plan X_0 the decomposition of all vectors $A_l \, (l = \overline{1,n})$ in this basis satisfies the condition:

$$\Delta_l = F_l - c_l \geq 0, \quad F_l = z_l x_l + v_l x_l + w_l x_l \quad (15)$$

plan X_0 is the optimal solution to the problem of linear programming.

ISSN 2306-4994 (print); ISSN 2310-8770 (online)

On the basis of the analysis of the estimates, it can be argued that the first reference plan of the problem of linear programming optimizing the structure of the insurance portfolio is not optimal, since not all estimates satisfy the optimality condition, that is, not all valuations are integral.

The next step in finding the optimal structure of the insurance portfolio is to improve the value of the target function, which in this case is equal to $0 \cdot SR + 0 \cdot PP + 0 \cdot SP - M \cdot 1 = -M < 0$, by changing the basis and transition to a new support plan.

Let's consider how, proceeding from the initial reference plan, proceed to the next supporting plan, which corresponds to the purposeful process of crossing the angular points of the solution multiplier.

Because $A_{n+1}, A_{n+2}, A_{n+3}, A_{n+4}$ is a basis of a 4-dimensional space, then each of the vectors of the relation can be decomposed by these vectors of the basis, and in the only way:

$$A_j = \sum_{i=1}^{4} x_{ij} A_i, \ \ j = 1,\ 2,\ ...,\ n. \tag{16}$$

Consider such a schedule for any non-free vector, for example, for A_1:

$$x_{n+1,1} A_{n+1} + x_{n+2,1} A_{n+2} + x_{n+3,1} A_{n+3} + x_{n+4,1} A_{n+4} = A_1. \tag{17}$$

Assume that in (18) there is at least one positive coefficient $x_{n+i,1}$.

Let's introduce some unknown value $\theta > 0$, we multiply both sides of it (18) and subtract the result from equality (16). We get it:

$$\begin{aligned} &(x_{n+1} - \theta \cdot x_{n+1,1})\, A_{n+1} + (x_{n+2} - \theta \cdot x_{n+2,1})\, A_{n+2} + \\ &+ (x_{n+3} - \theta \cdot x_{n+3,1}) A_{n+3} + (x_{n+4} - \theta \cdot x_{n+4,1})\, A_{n+4} + \theta \cdot A_1 = A_0. \end{aligned} \tag{18}$$

Vector $X_1 = (x_{n+1} - \theta \cdot x_{n+1,1};\ x_{n+2} - \theta \cdot x_{n+2,1};\ x_{n+3} - \theta \cdot x_{n+3,1}, x_{n+4} - \theta \cdot x_{n+4,1};\ \theta;\ 0, ..., 0)$ (19)

is a task plan in the event that its components are inalienable. By assumption $\theta > 0$, components of the vector X_1, which are included $x_{n+i,1} \le 0$, will be inalienable, therefore, it is necessary to consider only those components that contain positive ones $x_{n+i,1} (i = 1,\ 2,\ 3,\ 4)$. You need to find this value $\theta > 0$, for which everyone $x_{n+i,1} > 0$ operated condition inseparable plan objectives:

$$x_{n+i} - \theta \cdot x_{n+i,1} \ge 0. \tag{20}$$

From (20) we obtain that for the sought after $\theta > 0$ the condition must be fulfilled $\theta \le \frac{x_{n+i}}{x_{n+i,1}}$.

Vector X_1 will be a objectiv plan for any one θ, satisfying the condition:

$$0 < \theta \le \min_i \frac{x_{n+i}}{x_{n+i,1}}, \tag{21}$$

where we find the minimum for those i for which $x_{n+i,1} > 0$.

The reference plan cannot contain more than 4 positive components, so in the plan it is necessary to convert at least one of the components to zero. Suppose that $\theta = \theta^* = \min_i \frac{x_{n+i}}{x_{n+i,1}}$ for some value i, then the corresponding component of the plan X_1 converted to zero. Let it be the first component of the plan:

$$\theta^* = \min_i \frac{x_{n+i}}{x_{n+i,1}} = \frac{x_{n+1}}{x_{n+1,1}} \tag{22}$$

Replace the meaning θ^* in the expression (19):

$$(x_{n+1}-\frac{x_{n+1}}{x_{n+1,1}}\cdot x_{n+1,1})A_{n+1}+(x_{n+2}-\frac{x_{n+1}}{x_{n+1,1}}\cdot x_{n+2,1})A_{n+2}+ \\ +(x_{n+3}-\frac{x_{n+1}}{x_{n+1,1}}\cdot x_{n+3,1})A_{n+3}+(x_{n+4}-\frac{x_{n+1}}{x_{n+1,1}}\cdot x_{n+4,1})A_{n+4}+\frac{x_{n+1}}{x_{n+1,1}}\cdot A_1=A_0. \tag{23}$$

if ticked $x_{n+i}-\frac{x_{n+1}}{x_{n+1,1}}x_{n+i,1}=x_i'\ (i=\overline{2,4})$, $\frac{x_{n+1}}{x_{n+1,1}}=x_1'$, then the equation can be presented as:

$$x_{n+1}'A_{n+1}+x_{n+2}'A_{n+2}+x_{n+3}'A_{n+3}+x_{n+4}'A_{n+4}+x_1'A_1=A_0 \tag{24}$$

to which such a reference plan corresponds:

$$X_2=(x_1';\ 0;\ ...;\ 0;\ x_{n+1}';\ x_{n+2}';\ x_{n+3}';\ x_{n+4}') \tag{25}$$

To determine the next reference plan, it is necessary to continue the process in a similar way: any vector not included in the basis, decompose on the basis vectors, and then define the following $\theta^*>0$, for which one of the vectors is excluded from the basis.

Summarizing the considered process, we can state: the definition of new support plans is to choose a vector to be introduced into the basis, and the vector to be deduced from the basis. This procedure corresponds to the transition from one basis to another using the Jordan-Gauss method.

It should be noted that for a case where the vector A_1 be included in the base, and all his schedule $x_{i+n,1}\le 0$, obviously there is no such value $\theta>0$, which would exclude one of the vectors. In this case, the plan X_1 contains 1 positive component, hence the system of vectors $A_1, A_{n+1}, A_{n+2}, A_{n+3}, A_{n+4}$ will be linearly dependent and determines the non-cubic point of the solution multiplier. Functional cannot dial in it the maximum value. This means that the function is unbounded on the multithreading of solutions.

Consider the algorithm for changing the basis of the linear programming problem, which is accompanied by the transition to a new simplex table (new reference plan):

1) for those values of a string that does not satisfy the optimality condition, the most negative value is chosen, that is, one that has the greatest negative impact on the target function. Selected value of the estimated line and defines a column that shows which variable should be entered into the basis for improvement of the solution found;

2) the column of values is determined $\theta_1\div\theta_4$ (identifier of the basic variable to be derived from the basis) by dividing the plan into the corresponding elements of the column of the simplex table only if they are positive;

3) from the values obtained in the previous step $\theta_1\div\theta_4$, the minimum is selected, which determines the directional line, and hence the variable to be deduced from the basis;

4) at the intersection of the directional column and the row is determined by the solving element, which acts as the basis for the transition from this supporting plan to the new;

5) a new simplex table is formed and its verification for optimality is carried out.

In the new simplex table, first we fill in the first two columns «Basis» and «C_{bas}», and the rest of the elements of the new table are calculated according to the following rules:

– each element of the solving (guiding) line must be divided into a solvable element and the resulting numbers are written in the corresponding line of the new simplex table;

– the solving column in the new table is written as a unit with a unit instead of a solvable element;

– if there is a zero element in the guiding line, then the corresponding column is rewritten to the new simplex table unchanged;

– if there is a zero element in the guiding column, then the corresponding line is rewritten in a new table unchanged.

ISSN 2306-4994 (print); ISSN 2310-8770 (online)

All other elements of the next simplex table are calculated according to the rule of the rectangle.

To determine any element of a new table by this rule, in the previous simplex table make a conditional rectangle whose vertices are formed by the following numbers:

1 — solving element (number 1);

2 is the number standing at the place of the element of the new simplex table, which we have to calculate;

3 and 4 — elements that are placed in two other opposite vertices of a conditional rectangle.

The required element of a new simplex table is determined by the following formula:

$$\frac{\text{numeric}\,1\cdot\text{numeric}\,2-\text{numeric}\,3\cdot\text{numeric}\,4}{\text{soluble element}}. \quad (26)$$

Let's consider the described algorithm of formation of a new reference plan on an example of optimization of an insurance portfolio of the company.

$\min(F_l - c_l) = F_k - c_k = \Delta_k$, that is the minimum value is reached for k vector $1 \le k \le n$. Then the vector is included in the basis A_k. The corresponding column of the simplex table will have the name of the guide.

To get the coefficients of the vectors expansion A_0, A_1 ,..., A_{n+4}. According to the vectors of a new basis (the transition to the next support plan and the creation of a new simplex table), necessary: to divide all elements of the guideline line into the decoupling element; calculate all other elements according to the formulas of the full Jordan Gauss exceptions (rectangle rule).

Then you need to check the new values of the estimated line. If all $F_1 - c_1 \ge 0$, plan X_1 — is optimal, otherwise they will go to find the next support plan. The process continues until an optimal plan is arrived or the fact that there is no solution to the problem is established.

If the evaluation line of the last simplex table is an estimate F_1 — $c_1 \ge 0$ corresponds to a free (non-invisible) variable, this means that the linear programming problem has an alternative optimal plan. You can get it by choosing a decoupling element in the specified column of the table and by taking one step (one iteration) with a simplex method. As a result, we obtain a new reference plan, which corresponds to the same value of the functional as that of the previous plan, ie the functional reaches the maximum value at two points of the solution multiplier, and therefore this problem has an infinite set of optimal plans.

Conclusions. The result of the selection of all possible combinations of insurance services in the insurance portfolio of the company that meet the boundary conditions is to determine the acceptable variant in which the insurance company will reach the maximum level of competitiveness.

Based on the above methodology, we will conduct a quantitative assessment of the competitiveness of Ukrainian insurance companies. The representative sample will include insurance companies with the maximum geographical diversification, a wide range of insurance services, comparable insurance rates, and high ratings, namely: UNICIA, TAS AG, PZU Ukraine, ORANTA, AXA Insurance, INGO, ASKA, KRAINA, VUSO, SK «Knyazha». Determine the input parameters for carrying out a quantitative assessment of competitiveness on the basis of the above-mentioned methodology *Fig. 1—4*.

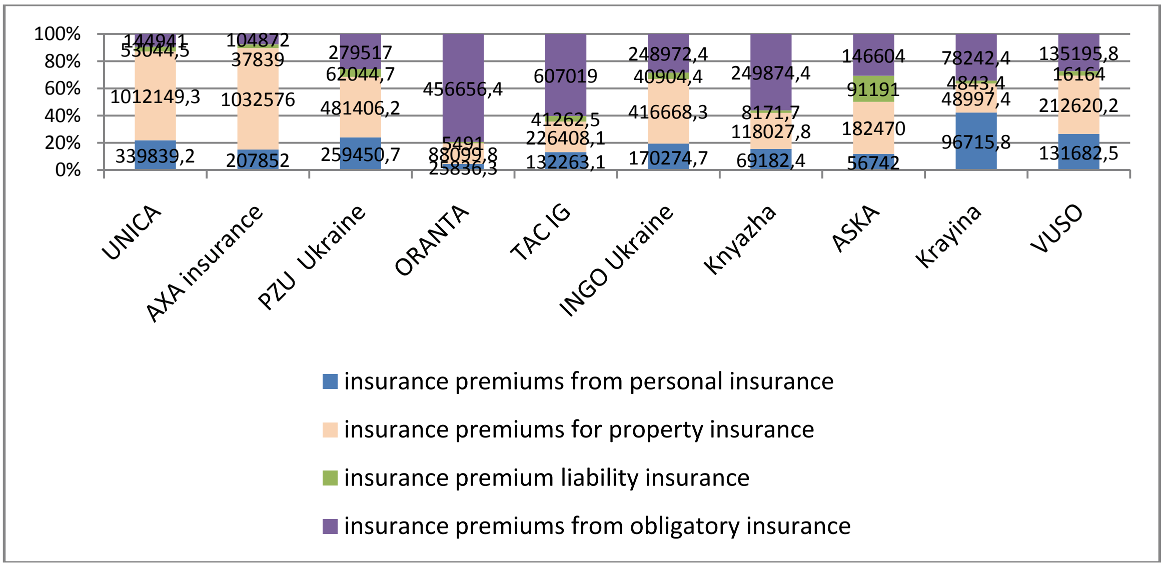

Fig. 1. **Components of the insurance portfolio**

Sources: compiled by the authors on the basis of (Insurance TOP 2018).

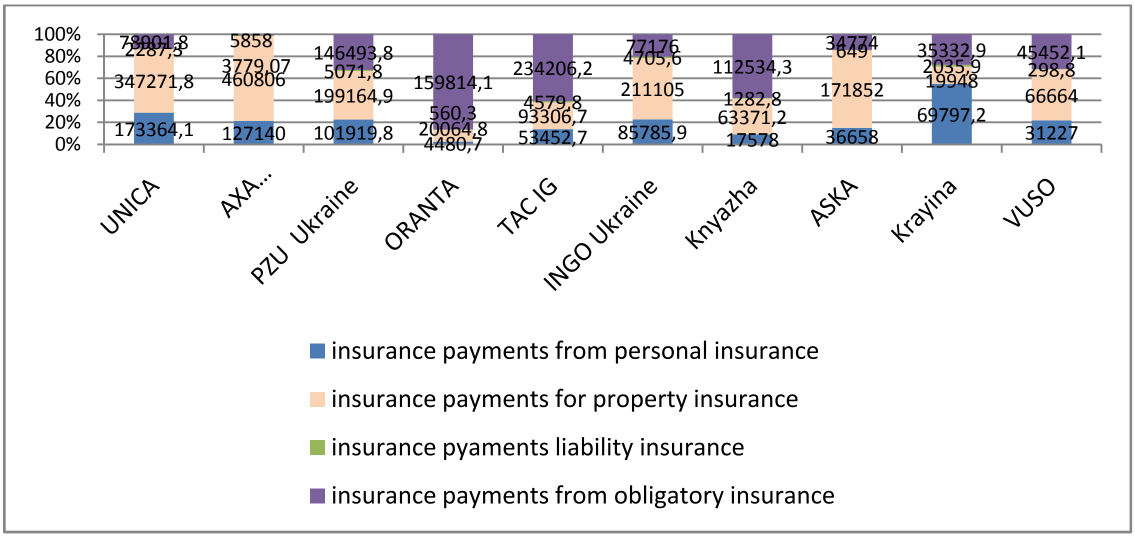

Fig. 2. **Component of the portfolio of insurance payments**

Sources: compiled by the authors on the basis of (Insurance TOP 2018).

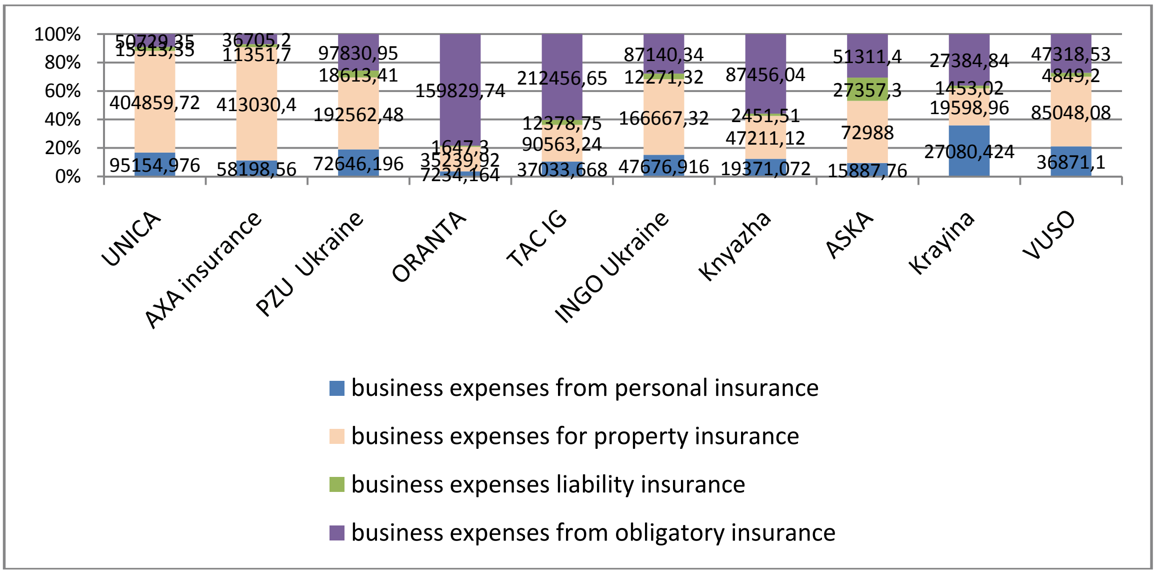

Fig. 3. **Components of the insurance business expenses**

Sources: compiled by the authors on the basis of (Insurance TOP 2018).

ISSN 2306-4994 (print); ISSN 2310-8770 (online)

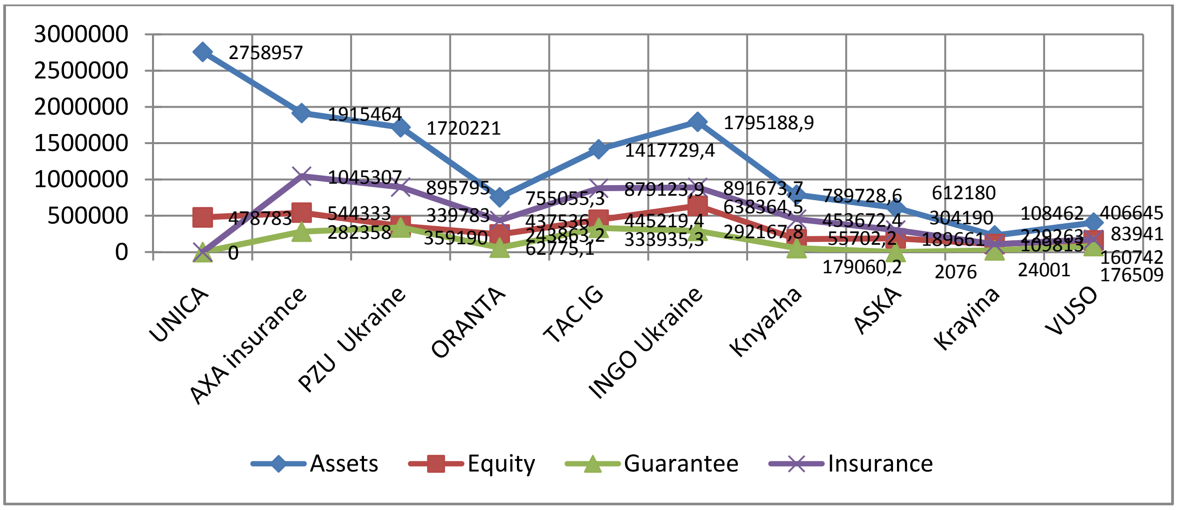

Fig. 4. **Financial indicators of insurers**

Sources: compiled by the authors on the basis of (Insurance TOP 2018).

According to the data given on the example of «Oranta» we solve the problem of linear programming of double simplex by a method with the definition of constraints and the transition to a canonical form of construction of the first reference plan (*Tabl. 1*).

Table 1

Extended matrix of system of constraints

0.048	0.153	0.095	0.793	0	0	0	1
4480.7	20064.8	560.3	159814	1	0	0	437536
25836.3	88099.3	5491	456656	0	1	0	8293169
7234.1	35239.9	1647.3	159830	0	0	1	576084

Sources: developed by the authors.

Let us introduce the system to a unit matrix by the Jordan-Gauss transformation method through the rectangle rule and obtain the following result.

Since there is a single matrix in the system, we take $X = (4,5,6,7)$ as the basis variables and express the basis variables through other ones:

x_4= -0.061x_1-0.193x_2-0.12x_3+1.262;

x_5= 5196.4x_1+10760.9x_2+18592.3x_3+235928.8;

x_6= 1815.3x_1-17.173x_2+49236.2x_3+7717092.3;

x_7= 2444.0x_1-4411.0x_2+17507.2x_3+374456.6;

Formulate the target function: $F(X) = 0.939x_1+0.807x_2+0.88x_3+1.262$

$0.061x_1+0.193x_2+0.12x_3+x_4=1.262$

$-5196.444x_1-10760.936x_2-18592.381x_3+x_5=235928.835$

$-1815.382x_1+17.173x_2-49236.286x_3+x_6=7717092.3$

$-2444.013x_1+4411.078x_2-17507.298x_3+x_7=374456.65$

When calculating the value Fc = 1.262 temporarily will not be considered.

Table 2

Matrix of the coefficient A = *a*(*ij*)

0.061	0.193	0.12	1	0	0	0
-5196.4	-10760.9	-18592.3	0	1	0	0
-1815.3	17.1	-49236.2	0	0	1	0
-2444.0	4411.0	-17507.2	0	0	0	1

Sources: developed by the authors.

Forecasting that free variables are 0, we get the first baseline:

$x_0 = (0,0,0,1.262,235928.835,7717092.3,374456.65)$

Table 3

Converting a simplex table

Basis	**B**	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_4	-15.113	0.167	0	0.885	1	0	0	0
x_5	1149425.1	-11158.6	0	-61301.8	0	1	0	2.44
x_6	7715634.5	-1805.8	0	-49168.13	0	0	1	-0.004
x_2	84.89	-0.554	1	-3.969	0	0	0	0
F(X0)	68.516	-1.387	0	-4.084	0	0	0	0
Basis	**B**	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_4	-15.113	0.167	0	0.885	1	0	0	0
x_5	1149425.1	-11158.6	0	-61301.8	0	1	0	2.44
x_6	7715634.5	-1805.8	0	-49168.13	0	0	1	-0.004
x_2	84.89	-0.554	1	-3.969	0	0	0	0
F(X0)	68.516	-1.387	0	-4.084	0	0	0	0

Sources: developed by the authors.

Table 4

The transformation of the simplex table by the Jordan-Gaussianmethod

Basis	**B**	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_7	345607.2	-3828.7	0	-20247.9	-22868.9	0	0	1	
x_5	306307.7	-1818.2	0	-11906.3	55789.3	1	0	0	
x_6	7716979.9	-1820.7	0	-49246.9	-89.03	0	1	0	
x_2	6.54	0.314	1	0.621	5.184	0	0	0	
F(X0)	5.279	-0.686	0	-0.379	4.184	0	0	0	
Basis	**B**	x_1	x_2	x_3	x_4	x_5	x_6	x_7	**min**
x_7	345607.2	-3828.7	0	-20247.9	-22868.9	0	0	1	-
x_5	306307.7	-1818.2	0	-11906.3	55789.3	1	0	0	-
x_6	7716979.9	-1820.7	0	-49246.9	-89.03	0	1	0	-
x_2	6.54	0.314	1	0.621	5.184	0	0	0	20.83
F(X1)	5.279	-0.686	0	-0.379	4.184	0	0	0	0

Sources: developed by the authors.

Table 5

Verification of the optimality criterion

Basis	**B**	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_7	425373.5	0	12196.2	-12670.1	40361.8	0	0	1
x_5	344188.0	0	5791.9	-8307.7	85817.1	1	0	0
x_6	7754912.7	0	5799.9	-45643.3	29980.2	0	1	0
x_1	20.833	1	3.185	1.979	16.515	0	0	0
F(X2)	19.572	0	2.185	0.979	15.515	0	0	0

Sources: developed by the authors.

Formation of function: $x_1 = 20.833$, $x_2 = 0$, $x_3 = 0$, $x_4 = 0$

max F(x) = 20.833.

Thus, calculations were made for all the parameters specified in the model and we obtain a quantitative indicator of the competitiveness of the insurance company «Oranta» F(x) = 20.833.Similarly, we will calculate the data from other insurance companies and get it as a result of the selection of all possible combinations of insurance services in the insurance portfolio of the company that meet the boundary conditions, the variant in which the insurance company will reach the maximum level of competitiveness and rank the insurance companies by this indicator.

Krainamax F(x) = 47.17;

AXAinsuarance max F(x) = 36.496 ;

 ISSN 2306-4994 (print); ISSN 2310-8770 (online)

VUSO max F(x) = 30.675;
UNICAmax F(x) = 29.24;
TACIG max F(x) = 24.39;
INGO Ukrainemax F(x) = 21.413;
ORANTAmax F(x) = 20.833;
PZU Ukrainemax F(x) = 17.452;
ASKAmax F(x) = 8.363;
Knyazhamax F(x) = 5.348.

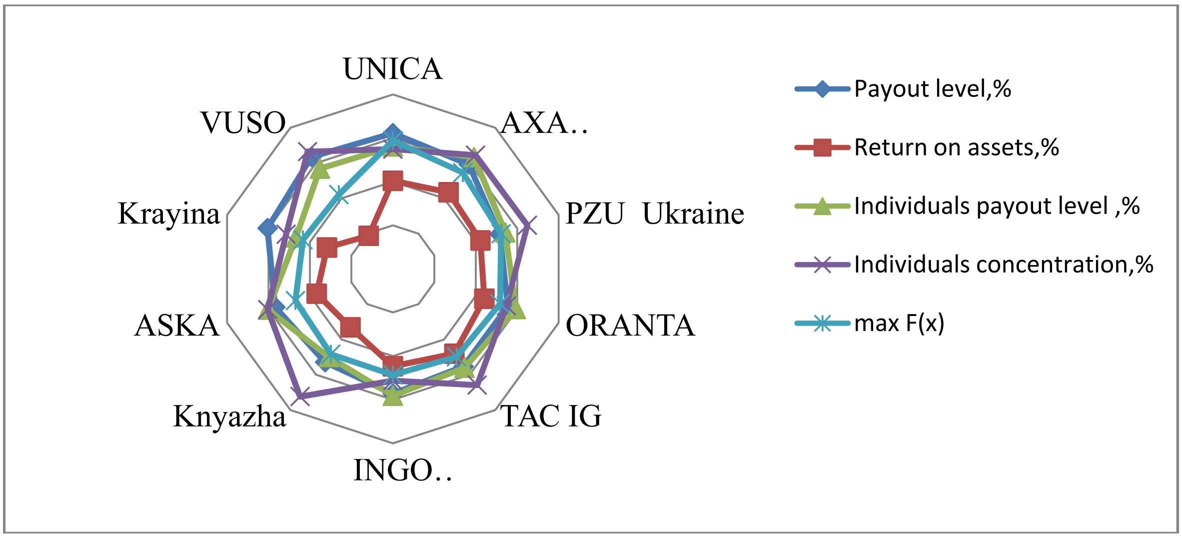

Fig. 5. **Quantitative representation of the concept «sustainable development» insurance companies**

Sources: developed by the authors.

A quantitative assessment of the competitiveness of insurance companies determines the prospects for their further development in the context of targeting the «sustainable development» line. In this research, the paradigm of «sustainable development» in the functioning of insurance companies based on their competitiveness implies the formation of an optimal result from the use of available resources and expansion of the markets for the sale of insurance services. It is the implementation of the noted concept reflected in *Fig. 5*, which describes the depiction of «sustainable development» of investigated insurance companies.

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Статтю рекомендовано до друку 31.08.2020 *© Д'яконова І. І., Кравчук Г. В., Шелюк А. А., Хабер Дж.*

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The article is recommended for printing 31.08.2020 *© Dyakonova I., Kravchuk A., Sheliuk A., Haber J.*

 ISSN 2306-4994 (print); ISSN 2310-8770 (online)