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### **ALGORITHMIZATION OF PROCEDURES FOR COORDINATION INDIVIDUAL OPINIONS IN THE PROCESS OF IMPLEMENTATION ORGANIZATIONAL AND MANAGERIAL LABOR RELATIONS**

**Abstract.** The methods of making a collective decision and choosing group matching procedures are analyzed. It is assumed that the methods of evaluation are given in the scales of qualitative order, including the binary relations of alternatives. The analysis of group selection procedures in the management of complex systems is carried out. The necessary preliminary conditions for the construction of mathematical models of group choice in the management of complex systems are systematized. The sequence of solving the problem is to analyze the conditions for the harmonization of the rules of group choice, requirements and serial relations, from the point of view of reasonableness in a group choice, algorithmization of procedures for the coordination of individual opinions.

The development of mathematical models of formation of group decisions in different classes of relations is proposed. This pursued the goal of providing the «subtleties» of group choices. As rules of outgoing (basic) rules, rules of the type of majority apply. Repeated appeal to individuals involves clarifying their thoughts on the alternatives included in the group of the best. As a result, majoritarian relations were obtained and their generalizations, which express the general opinion of the whole group of experts.

The formalization of interactive procedures for consistently obtaining the group consensus of expert opinions provided that the relations are transitive, including feedback on the convergence of opinions. The selectivity of the choice function is achieved by expanding the structure of the family of solving sets. The result of this formalization is the refinement of expert opinions on the alternatives included in the group of the best, but with a minimum number of votes. This formalization creates the necessary conditions for the processing of results in decision support information systems.

Algorithmization of procedures for coordinating individual thoughts in the management of publishing. The proposed algorithm allows to account for the initial opinions of all experts and customers of the publishing house in the form of a profile of individual selection functions. As a result, a sequence of steps has been formed to reconcile individual thoughts in the management of publishing.

**Keywords:** binary relations, group choice, legal organization of publishing house management, organizational and managerial labor relations, expert assessments.

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## АЛГОРИТМІЗАЦІЯ ПРОЦЕДУР ПОГОДЖЕННЯ ІНДИВІДУАЛЬНИХ ДУМОК У ПРОЦЕСІ РЕАЛІЗАЦІЇ ОРГАНІЗАЦІЙНО-УПРАВЛІНСЬКИХ ТРУДОВИХ ПРАВОВІДНОСИН

**Анотація.** Аналізуються економіко-правові методи ухвалення колективного рішення і вибору процедур групового узгодження у процесі реалізації організаційно-управлінських трудових правовідносин. Передбачається, що методи оцінок наводяться у шкалах якісного порядку, включаючи бінарні відносини альтернатив. Проведено аналіз процедур групового вибору при управлінні складними системами. Систематизовано необхідні попередні умови побудови економіко-правових моделей групового вибору при управлінні складними системами. Послідовність розв'язання поставленої проблеми передбачає аналіз умов узгодження правила групового вибору, вимог і порядкових відносин щодо розумності у груповому виборі, алгоритмізації процедур узгодження індивідуальних думок працівників.

Запропоновано розроблення економіко-правових моделей формування групових рішень у різних класах правовідносин. При цьому переслідувалася мета забезпечення «тонкощі» групового вибору. Як вихідні (базові) правила застосовуються правила типу більшості. Повторне звернення до працівників передбачає уточнення їхніх думок щодо альтернатив, що входять у групу кращих. У результаті отримано мажоритарні відносини та їх узагальнення, які виражають загальну думку всієї групи експертів.

Проведено формалізацію інтерактивних процедур послідовного отримання групової узгодженості думок експертів за умови транзитивності правовідносин, включаючи зворотний зв'язок зі збіжності думок. Селективність функції вибору досягається шляхом розширення структури сімейства вирішальних множин. Результатом цієї формалізації є уточнення думок експертів щодо альтернатив, які входять у групу найкращих, але з мінімальним числом голосів. Ця формалізація створює необхідні умови обробки результатів в інформаційних системах підтримки ухвалення рішень працівниками.

Виконано алгоритмізацію процедур узгодження індивідуальних думок у процесі реалізації організаційно-управлінських трудових правовідносин у видавництві. Запропонована алгоритмізація дозволяє здійснювати облік вихідних думок усіх роботодавців на видавництві у формі профілю індивідуальних функцій вибору. У результаті сформовано послідовність кроків узгодження індивідуальних думок у процесі правової організації праці видавництва.

**Ключові слова:** бінарні відносини, груповий вибір, правова організація праці, організаційно-управлінські трудові правовідносини, експертні оцінки.

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**1. Introduction.** Decisions made by the employer under conditions of uncertainty, and goals formed in the context of the expected results, include economic and legal procedures for coordinating the opinions of specialists on many criteria. The guaranteed result of group decision-making involves the harmonization of the opinions of the majority of participants of the choice in accordance with legal and regulatory instruments, local acts of the enterprise. However, this reference point, approved by the labor collective of specialists and experts, is an element of the information and legal space that is useful for the decision maker, including the employer. Moreover, a formal mathematical analysis cannot give a rigorous and accurate result of the choice of alternatives under conditions of uncertainty. A variety of types of expert information and economic and legal procedures for obtaining a group solution determine certain methodological problems of a general nature that one has to face when organizing and conducting any group choice. One of the key problems that arise during the economic and legal examination is the analysis of the individual judgements of each expert, the aggregation of the collective opinion of workers to find a final solution.

The complexity of this problem lies in the fact that the class of complete transitive legal relations does not always correlate with the monotonous rules of group coordination. When analyzing the selected subsystem for coordinating decisions on a variety of alternatives, it is necessary to take into account its relationship with the rest of the more complete system of making organizational and managerial decisions in the field of labor law. Without the ability and means to accurately describe all these relationships, either the representation of the decision maker or the opinion of experts who have these representations is used.

The listed aspects are especially relevant for the labor processes of managing hierarchical structures of the employer. In particular, such decision-making procedures as organizing working hours, structuring local regulations, employer workflow, selecting prepress options, creating multimedia information content, evaluating the effectiveness of using information systems, require the coordination of individual expert opinions.

Usually, decisions on managing the organization of labor and publishing are made taking into account labor law norms (including the norms of working hours outlined in Articles 50—53, 67, 73 of the Labor Code of Ukraine), the interests of workers and the employer. In some cases, the subjective preferences of experts, taking into account experience and intuition, come first. But this kind of subjective approach is acceptable only in conditions of small-scale publishing projects with a limited circle of potential consumers. When trying to go beyond these restrictions to expand the scope of projects and increase the efficiency of the process of legal organization and management of the publishing house, objective difficulties arise. These difficulties are associated with the lack of economic and legal tools that can optimize decision-making procedures.

Overcoming these difficulties should be carried out by algorithmizing the procedures for harmonizing individual views in the management of publishing. The central link of this methodology is the interactive group selection procedures, which allow optimize the economic and legal decision-making process by the employer.

Solving the problem of the algorithmizing of the procedures for harmonizing individual views in the process of organizing and managing a publishing house will provide the current economic and legal tools for improving the legal organization of labor, information support for the publishing and printing industries. In particular, the legal organization of labor can be increased, the labor responsibilities of workers clarified, and the legal mechanism for protecting labor rights improved. As a result, the productivity of the technological process of creating publishing products will be increased and its cost will be reduced.

**2. Literature data analysis and problem statement.** There is a number of approaches that make it possible to exclude obviously unacceptable options, limit the many analyzed alternatives, improve the legal regulation of organizational and managerial relations and the mechanism for their legal protection. So [1], the use of high-quality scales for measuring estimates of alternative solutions is proposed. This is quite justified, since a lot of evidence has already accumulated that there are general characteristics of the behavior and interaction of a group of people in the

formation of a common solution for various areas of activity. However, in the course of this measurement of assessments, the coordination of expert opinions is ignored.

The study [2] considers the problems of group decision making. Special attention is paid to the structuring of the stages of making managerial decisions, but at the same time, non-manipulative approval procedures have not been analyzed.

When managing organizational and socio-economic systems, a large amount of information is of a qualitative nature, therefore, issues of qualitative analysis are essential. The work [3] is devoted to these issues, in which at the same time there is no numerical interpretation of the qualitative analysis performed.

Features of decision making using the theory of expectations are analyzed in a scientific article [4]. This article pays attention to the design of interactive procedures for the formation of group decisions. However, the authors overlook the question of finding the median in a particular class of preference relations or choice functions.

Empirically reasonable decisions of group choice are given in the study [5]. This study identifies the prospects for the development of empirical decision making technologies. At the same time, the authors do not take into account non-manipulative procedures for coordinating decisions.

An empirical analysis of dynamic solutions is reflected in [6]. The main aspects of the dynamic decision algorithmization are analyzed. However, no attention is paid to the analysis of the conditions for agreeing on a group choice rule, requirements and order relations, from the point of view of rationality in group choice.

The article [7] is devoted to the problems of a multilevel analysis of similarity in solutions with a hidden profile. A hierarchical decision-making model is built. At the same time, in this model there are no scales for qualitative or quantitative estimates.

The study [8] analyzes the antecedents for decision making. Methods for achieving quality and flexibility in managing an innovation portfolio are suggested. At the same time, the formed judgements and conclusions are informal in nature. Consequently, these judgements and conclusions cannot be further processed in the corresponding decision support information system.

The decision-making technique in conditions of risk and uncertainty is presented in [9]. However, there is no formalization of the procedure for sequentially obtaining group consistency of expert opinions, provided the relations are transitive.

The development of a multifactor authentication method for the algorithmization of complex technical systems is proposed in [10]. At the same time, special attention is paid to information security issues based on hybrid cryptocode designs. But it is not paid attention to the problems of making managerial decisions.

The implementation of group choice methods in the theory of system constraints was proposed in [11]. However, there is no consideration of the synthesis and analysis of group selection procedures according to axiomatically formulated conditions that determine individual and group opinions.

The study [12] analyzes the general issues of publishing management. In particular, issues of decision-making are considered in the context of the need for harmonization of views. However, there is no algorithmization of opinion coordination procedures.

The study [13] reveals the features of legal regulation of certain types of organizational and managerial relations in labor law. The experience of the USA, Canada, Latin America, and the EU in this area is separately disclosed.

The study [14] analyzes the conservation and protection of labor rights in the field of organizational and managerial relations. In this case, special attention was paid to the mechanism of preventive conservation and protection, labor rights, methods and means of such protection and their impact on organizational and managerial legal relations.

Thus, the analysis of scientific research indicates the absence of a holistic scientifically based algorithmization of the procedures for harmonizing individual views in the process of publishing management in considered works.

**3. The purpose and objectives of the study.** The aim of the work is to develop a methodology for the legal organization of labor and management in a publishing house based on the coordination of individual opinions of experts. This will provide an opportunity to improve organizational and managerial legal relations, a mechanism for protecting labor rights, and information support for publishing through the development of a mathematical apparatus for supporting managerial decision-making.

To achieve this goal, the following tasks were solved:

- group selection procedures are analyzed when managing complex systems;
- developed mathematical models for the formation of group decisions in various classes of relations;
- formalized interactive group selection procedures.

**4. Development of algorithms for the harmonization of individual opinions in the process of publishing management**

**4. 1. Analysis of group selection procedures in the implementation of organizational and managerial legal relations, publishing management**

In this work, the object of management is the organizational and managerial legal relationship in the publishing house, a set of prepress, printing and post-printing processes for the production of printing products.

Publishing processes, some organizational and managerial relations, for the most part, are poorly formalized. In particular, such decision-making tasks as the selection of printing equipment, materials and production technology of the publication, the selection of prepress software, the development of the design of publications, the selection of trapping parameters for their solution, ensuring equal rights for workers, and increasing labor efficiency require the involvement of experts.

The impossibility of numerically determining decision-making parameters for publishing management dictates the need to use a group selection methodology, which forms the corresponding expert procedure for a real object.

The basis of the proposed model approach is the use of a cybernetic model of a control system (CS) by a publishing house. This control system includes a passive (controlled) and active (control) system, as well as a control loop and a correction loop of a block of models of a controlled system.

The control system includes directives of the labor management system for optimizing publishing performance. The managed system includes a set of mathematical models of algorithms for the harmonization of individual opinions in the process of organizing labor and management in the publishing house. The scheme of the CS model is shown in *Fig.*

It should be understood a set of actions to achieve acceptable for the publisher and the team values of performance indicators and a qualitative assessment of their «significance» under the management in the context of this work.

Monitoring of the legal system of labor organization and publishing management should be carried out systematically taking into account the influence of the market environment and labor protection. The quality of the legal organization of labor and the management of the publishing house depends on the decision-making situation by the management in constantly changing working conditions both on the printing services market and inside the publishing house.

Factors of the legal organization of labor, publishers' controllability depend on the system of internal components and adaptability to changes in the external environment.

The interests of customers, employees, characteristics of the means of production: printing equipment and materials, characteristics of the software used, etc. can serve as input parameters for publishing management (see *Fig.*).

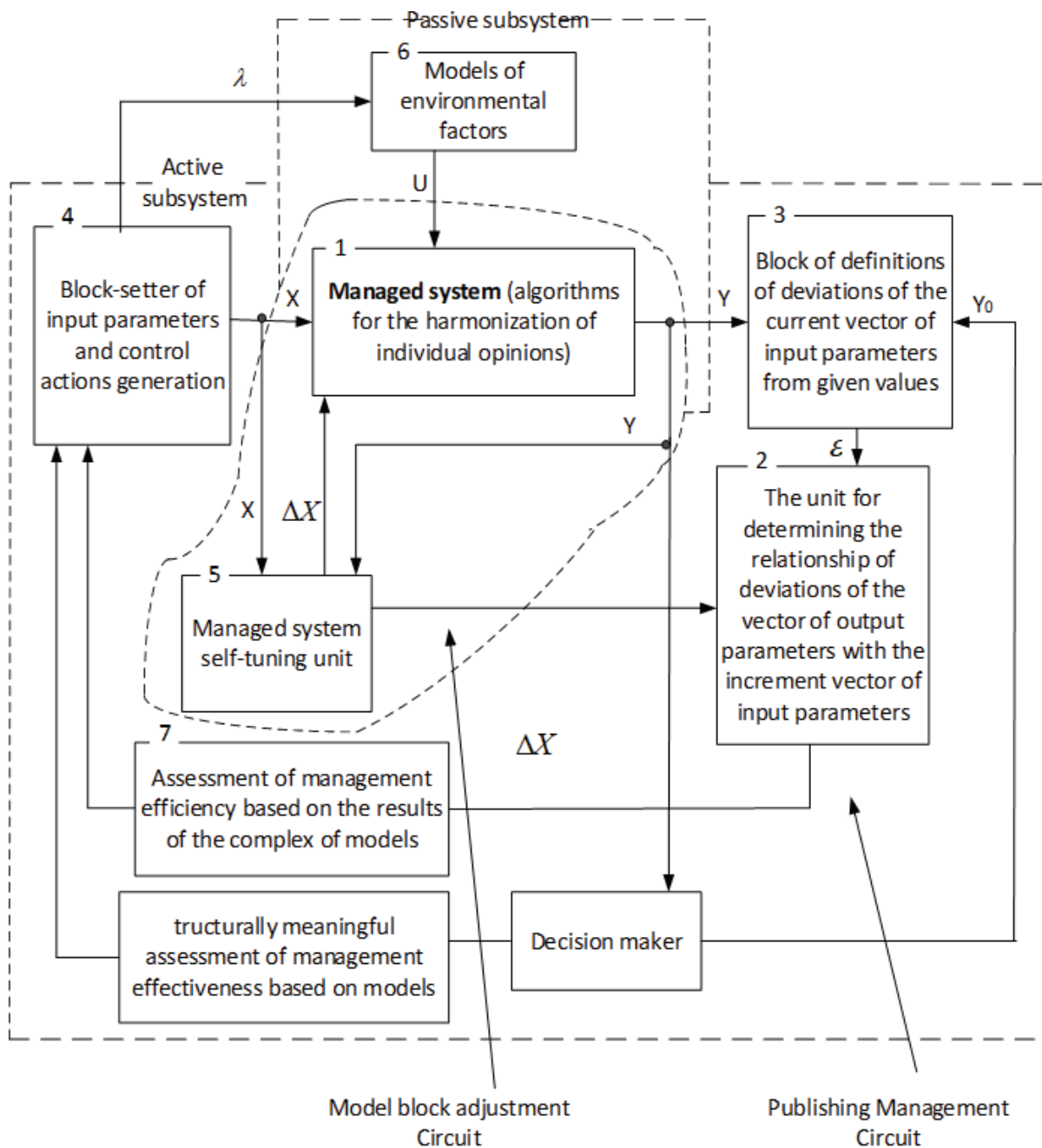


Fig. Conceptual model of a publishing management system

In Fig. the following conventions are accepted:

$\varepsilon$  is the vector of deviations of the publishing management parameters;  $\varepsilon = Y - Y_0$ ;

$U$  is the vector of the effects of environmental factors on the processes of publishing;

$\lambda$  is the vector of influence on environmental factors as a result of the coordination of individual opinions in the process of publishing management;

$Y_0$  — planned values of publishing management parameters;

$Y$  — actual values of publishing management parameters;

$X$  is the vector of technical and commercial capabilities and preferences of customers;

$\Delta X$  — adjusted values of the management criteria, based on the technical and production capabilities and preferences of customers.

The output parameters of the publishing management are the values of the resulting indicators of labor efficiency, the price of publishing products, the color scheme of the publication, the parameters of trapping, etc., agreed with experts.

The management criterion in this case is the economic and legal assessment of the effectiveness of the proposed expert procedures as part of a comprehensive model of the publishing management system.

The quality of management is formed under the influence of the preparatory stage of creating a management system. Analysis and the formation of a hierarchy of assessments and subsequent control should be carried out at the beginning of the stage of systematization of management structures. It is very important to determine the objectivity (subjectivity) of the goals of the publishing house, the ability to form guidelines.

The modeling tasks are traditionally based on the search for the structural and functional dependence  $Y = f(X, \alpha)$ , where  $Y$  is the vector of output variables,  $X$  is the vector of input parameters,  $\alpha$  is the vector of model parameters. The construction of such a single closed model is impossible for poorly formalized publishing management tasks. Such difficulties are eliminated by approximating a weakly formalized problem by a multitude of smaller formalized problems for which local formal models can be constructed.

Among a large number of methods for the analysis and synthesis of complex control systems, of considerable theoretical and practical interest are those that take into account the behavior of the most important element of any control system-man and his work. In the framework of the collective interaction problem, essentially different aspects of this problem are investigated. One of these aspects is the problem of group choice.

In many management tasks, the situation necessitates the mandatory choice of one or more alternatives. In particular, in the problems of publishing information support, there is a need to form a list of evaluation parameters that affect the development efficiency [15]. It is also advisable to use an expert approach to the problems of publishing management with a combination of cluster, multifactor, and discriminant analysis tools [16].

To solve the problem of harmonizing individual views in managing the publishing house, we will use the rule of a simple majority. Although this rule classically refers to the task of voting, and not to the task of expert choice, its use in the context of this work is explained by the fact that the theory of group choice is the methodological basis of expert assessments.

From all possible situations of group choice let's distinguish a situation that satisfies the following conditions:

- individuals making a choice must have a certain «freedom of choice», that is, there are several alternative plans for further behavior, the choice among which depends solely on the will of these individuals (for example, the labor collective, employer);
- individuals making a choice must understand that there is no single, objective criterion of choice in this situation, and they require a creative, and not a «mechanical» analysis of the economic and legal situation associated with the analysis of their own relations to it;
- individuals making a choice should be interested in the end results of that choice;
- in situations of choice, the full totality of alternatives to choice is created from the outside and is fully understood by individuals making the choice;
- a group of individuals making a choice is formed from the outside, before the implementation of the selection process.

The last two conditions draw a certain line between the selection processes and decision-making processes in the legal organization of labor. These conditions exclude from consideration more and more complex forms of behavior related to decision-making processes associated with determining the initial set of alternatives, identifying the need for decision-making, etc.

Possible areas of application of group selection procedures in publishing management are:

- scientific and technical forecasting;
- production management of printing products;
- modeling of technological processes of publishing;
- publishing planning;
- design of new equipment;
- assessment of labor efficiency;

- modeling of the local legal organization of labor;
- assessment of product quality.

The mathematical models of group choice when managing complex systems are distinguished by two important features:

- the content of the concept of «compromise» depends on the specific formulation of the problem, i.e. in one case, for example, it is advisable to use a mathematical approach, in the other case an axiomatic;
- with the fixed principle of compromise, it is quite possible that for non-manipulative approval procedures group choice will be empty or trivial.

However, for a wide class of group choice rules, the conditions of selectivity and efficiency are incompatible with the condition of the absence of a dictator. This substantially narrows the class of rules that guarantee the «constructiveness» of group choice.

In relation to the applied area of publishing management, formalization of group selection procedures creates opportunities for dual training based on relevant information systems [17].

#### 4.2. Formation of group decisions in various classes of relations

The construction of transitive group orderings of objects is one of the main questions in the theory of group choice. Of particular interest is the construction of an algorithm for finding transitive modifications of majority relations, as well as relations of a more general form, in the case when individual relations of preference, individual labor relations are linear quasiorders.

To formalize the object of study, the classical theory of binary relations should be applied. To this end, it is necessary to give the basic definitions necessary for the algorithmization of the procedures for harmonizing individual views in the process of publishing management.

There are situations of choice when experts are not able to measure their advantages in quantitative measurement scales. Therefore, in the process of algorithmizing the procedures for harmonizing individual views for a group of experts, it is advisable to determine their estimates in the form of binary relations for the indicators.

Let  $M$  be an arbitrary set of compared objects,  $M \times M$  is Cartesian square. As usual, by a binary relation between elements of  $M$  we mean an arbitrary subset of a set  $M \times M$ . The fact that the elements  $a, b \in M$  are in a binary relation  $R$  is denoted as follows:

$$(a, b) \in R \text{ or } aRb .$$

Let  $\Delta$  denote the diagonal of the set  $M \times M$ , i.e....

$$\Delta = \{(a, a), a \in M\} . \tag{1}$$

The product  $PR$  of the binary relations  $P$  and  $R$  is defined as follows:

$$(a, b) \in PR \Leftrightarrow \exists x \in M, (aPx) \wedge (xRb) . \tag{2}$$

For any  $R$ , we define the inverse ratio  $R^{-1}$  as follows:

$$(a, b) \in R^{-1} \Leftrightarrow (b, a) \in R . \tag{3}$$

A complement to the relation  $R$  is such a relation  $\bar{R}$  that

$$(a, b) \in \bar{R} \Leftrightarrow (a, b) \notin R . \tag{4}$$

A binary relation  $R$  is called reflexive if  $R \supseteq \Delta$ , or, in other words

$$\forall a \in M: (a, a) \in R . \tag{5}$$

A relation  $R$  is called transitive if  $R^2 \subseteq R$ , i. e

$$\forall x, y, z \in M (xRy) \wedge (yRz) \Rightarrow xRz . \tag{6}$$

The relation  $R$  is called linear if  $R \cup R^{-1} = M \times M$ . In other words, a relation is called linear by which any two objects from are comparable, i. e.

$$\forall a, b \in M: ((a, b) \in R) \vee ((b, a) \in R) . \tag{7}$$

A relation  $R$  is called quasiorder if it is reflexive and transitive.

Let a relation  $R$  be given. Its transitive closure  $R^\tau$  is the smallest transitive relation containing  $R$ . It is easy to show that  $R^\tau$  coincides with the intersection of all transitive relations

If  $R$  is the second majority relation, then its modification is the relation constructed as follows:

- 1) if  $n(a, b) > n(b, a)$ , then  $(a, b) \in R'$ ;
- 2) if  $n(a, b) = n(b, a)$ , then  $aR'b$  or  $bR'a$ ;
- 3) if  $n(a, b) < n(b, a)$ , then  $(a, b) \notin R'$ .

The above definitions of modifications are related as follows. We first consider the generalized majority relation (in particular, of course, the rules of the simple majority and the  $\alpha$ majority). Let us take such a relation:  $aPb \Leftrightarrow n(a, b) > f(n)$ . Then, firstly,  $P$  will be the intersection of all modifications  $R$ , and secondly, the concept of modification of the relation  $R$  and modifications of the pair  $(R, P)$  for  $f(n) \leq \frac{n}{2}$  will coincide.

In the case of the second majority relationship, we take as  $P$  such a ratio  $aPb \Leftrightarrow n(a, b) > n(b, a)$ . And in this case, both definitions of the transitive modification will coincide and a match will be executed:

$$P = (R / R^{-1}) \cup \Delta. \tag{14}$$

The main task of further considerations is to construct an effective algorithm for finding modifications of pairs  $(R, P)$  that are transitive relations (transitive modifications), and to establish conditions for a pair  $(R, P)$  under which transitive modifications exist. To solve this problem, we need some notation and auxiliary facts.

Let  $a, b$  be arbitrary elements of the set  $M$ . We introduce the following notation:

$$\text{Tab} = \Delta \cup (a, b). \tag{15}$$

**Lemma.** Let  $A$  be an arbitrary quasiorder,  $a$  and  $b$  — any. Then:

$$(A \cup \text{Tab})^\tau = A\text{Tab}A. \tag{16}$$

*Evidence.* We show first that the  $A\text{Tab}A$  relation is a quasiorder. Indeed,  $\Delta \subseteq A\text{Tab}A$  where the  $A\text{Tab}A$  relationship reflexivity comes from. Let us prove that  $A\text{Tab}A$  is transitive.

Let  $(x, y) \in A\text{Tab}A$  and  $(y, z) \in A\text{Tab}A$ . Then there are such  $\alpha, \beta, \chi, \delta \in M$ , that

$$\begin{aligned} &(xA\alpha) \wedge (\alpha\text{Tab}\beta) \wedge (\beta Ay), \\ &(yA\chi) \wedge (\chi\text{Tab}\delta) \wedge (\delta Az). \end{aligned} \tag{17}$$

From  $\alpha\text{Tab}\beta$  follows  $\alpha = \beta$  or  $\alpha = a$ ,  $\alpha = a$ .

In the first case  $xA\chi$ ,  $\chi\text{Tab}\delta$ ,  $\delta Az$ , i. e.  $(x, z) \in A\text{Tab}A$ . Similarly  $(x, z) \in A\text{Tab}A$ , if  $\chi = \delta$ .

Now suppose that  $\alpha = \chi = a$ ,  $\beta = \delta = b$ . Then  $xAa$ , whence, i.e., the relation is transitive.

Due to reflexivity  $A \cup \text{Tab} \subseteq A\text{Tab}A$ , where  $(A \cup \text{Tab})^\tau \supseteq (A \cup \text{Tab})^3$ . On the other hand,  $(A \cup \text{Tab}) \supseteq (A \cup \text{Tab})^3 = A^3 \cup A^2\text{Tab} \cup \dots \supseteq A\text{Tab}A$ . Thus,  $(A \cup \text{Tab})^\tau = A\text{Tab}A$ , which was to be demonstrated.

**Theorem.** Let the ratio  $R \subseteq M \times M$  is linear, and ratio  $P \subseteq R$  retains strict preferences from  $R$ . Suppose that  $S$  is a transitive relation such that  $P \subseteq S \subseteq R$ ,  $(a, b) \in R$  and  $(b, a) \notin S$  for some objects  $a, b \in M$ . Then  $(S \cup \text{Tab})^\tau \subseteq R$ .

*Evidence.* Assume the opposite. Let  $(x, y) \notin R$ ,  $(x, y) \in (S \cup \text{Tab})^\tau$  for some objects  $x, y \in M$ . According to the lemma  $xS\alpha$ ,  $\alpha\text{Tab}\beta$ ,  $\beta Sy$  for some  $\alpha, \beta \in M$ . If  $\alpha = \beta$ , then transitivity implies  $(x, y) \in S \subseteq R$  contrary to the assumption. Therefore  $\alpha \neq \beta$ , and, means  $\alpha = a$ ,  $\beta = b$ .

Hence  $xSa$  and  $bSy$ . If there was a  $bSx$ , then from the transitivity of the relationship was  $bSa$ , which contradicts the condition. That is why  $(b, x) \notin S$ .

Because  $(x, y) \notin R$ , then from condition 2) we obtain  $yPx$ . Since  $P \subseteq S$ , then  $ySx$ . But on the other hand  $bSy$ , where  $bSx$ . We got a contradiction, therefore  $(S \cup \text{Tab})^\tau \subseteq R$ , as required.

We now describe the algorithm for finding the transitive modification of the pair  $(R, P)$ . At the first step we find a transitive closure  $P^\tau$ . Recall that the transitive closure of a relation  $P$  is a relation  $P^\tau$  equal to the intersection of all transitive relations including  $P$ , or, which is the same:

$$P^\tau = \bigcup_{k=1}^{\infty} P^k, \tag{18}$$

Where  $P^k = P \cdot P^{k-1} \cdot \dots \cdot P$  ( $k$  times).

The procedure for finding a transitive closure reduces to repeatedly raising to the power of the relation  $P$ . A sign of obtaining a transitive closure of the relation  $P$  is the equality:

$$P^k = P^{k-1}.$$

Note that, by definition,  $P$  is contained in the intersection of all modifications of the pair  $(R, P)$ . Therefore  $P^\tau$  is contained in the intersection of all transit modifications. Therefore, if  $P^\tau \supseteq R$ , then there are no transit modifications of the pair  $(R, P)$ . Otherwise, we denote  $Q_1 = P^\tau$  and go to the second step.

Suppose that at the  $i$ -th step ( $i \geq 1$ ) a transitive relation  $Q_i$  is formed, such that  $P^\tau \subseteq Q_i \subseteq R$ . If this relation is linear, then it is the desired transitive modification. Otherwise, there are objects such as  $a, b \in M$ , that  $a\overline{Q_i}b$  and  $b\overline{Q_i}a$ . Since the relation  $R$  is linear, then, for example,  $aRb$ . Therefore, we can go to the  $i + 1$ th step by setting  $Q_{i+1} = (Q_i \cup Tab)^\tau$ . According to the theorem, the relation  $Q_{i+1}$  is contained in  $R$ , while being different from  $Q_i$ .

The following necessary and sufficient condition for the existence of transitive modifications convenient for practical applications, as well as other consequences, is directly obtained from the theorem and algorithm.

*Consequence 1.* A transitive modification of a linear group relation  $R$  exists if and only if  $P^\tau \subseteq R$ .

*Consequence 2.* If  $f(n) < \frac{n}{2}$  and  $P^\tau \subseteq R$ , then  $P^\tau$  is a transitive modification.

*Consequence 3.* For a linear group relation  $R$ , if and only if there is a subset  $P$  that allows us to construct a transitive modification when  $R \supseteq [(R / R^{-1}) \cup \Delta]^\tau$ .

It is easy to see that relations satisfying the last condition must also satisfy the condition:

$$aRb \wedge b\overline{R}a \wedge bRc \wedge c\overline{R}b \Rightarrow aRc, \tag{19}$$

for any  $a, b, c \in M$ . It seems interesting to study group relations that satisfy this conditionally, and, in particular, build an analogue of transitive modifications for them.

We give an example illustrating the operation of the proposed algorithm. Let 4 experts have the following individual relationships on a variety of objects  $M = \{1, 2, 3, 4, 5\}$ :

$$R_1: 1 = 2 = 3 > 4 > 5,$$

$$R_2: 4 > 5 > 1 > 3 > 2,$$

$$R_3: 3 > 2 > 5 > 4 > 1,$$

$$R_4: 5 > 1 > 4 > 3 > 2.$$

Put  $f(n) = \frac{n}{2}$ . Then the relations  $P$  and  $R$  are given by the matrix:

$$P = \begin{pmatrix} I & I & I & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & I & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ I & 0 & 0 & 0 & I \end{pmatrix} \quad R = \begin{pmatrix} I & I & I & I & 0 \\ I & I & 0 & I & I \\ I & I & I & I & I \\ I & I & I & I & I \\ I & I & I & I & I \end{pmatrix} \tag{20}$$

The transitive closure of the relation  $P$  coincides with its square, so:

$$Q_1 = P^\tau = P^2 = \begin{pmatrix} I & I & I & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & I & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ I & I & I & 0 & I \end{pmatrix} \tag{21}$$

In the second step, we can add the pair (4,1) to  $Q_1$ , since  $(4,1) \notin Q_1$  and  $(1,4) \notin Q_1$ . We get:

$$Q_2 = (Q_1 \cup (4,1))^\tau = \begin{pmatrix} I & I & I & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & I & I & 0 & 0 \\ I & I & I & I & 0 \\ I & I & I & 0 & I \end{pmatrix} \tag{22}$$

In the third step, connect the pair (5,4):

$$Q_3 = (Q_2 \cup (5,4))^\tau = \begin{pmatrix} I & I & I & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & I & I & 0 & 0 \\ I & I & I & I & 0 \\ I & I & I & I & I \end{pmatrix} \tag{23}$$

The resulting relation  $Q_3$  is linear, and, therefore, is the desired transitive modification of the relation  $R$ :

$$R' = Q_3 : 5 > 4 > 1 > 3 > 2.$$

We note some features of the algorithm.

*Remark 1.* When finding a transitive modification on a computer, it is convenient to calculate the relation matrix  $Q_i = (Q_{i-1} \cup \Delta(a, b))^\tau$  using the lemma. If  $Q_i = (q_{\alpha\beta}^i)^m_{\alpha\beta} = 1, \Delta(a, b) = (\delta_{\alpha\beta})^m_{\alpha\beta} = 1$ , then from the relation we get:

$$q_{\alpha\beta}^i = \sum_{x,y=1}^m q_{\alpha x}^{i-1} \delta_{xy} q_{y\beta}^{i-1} = \sum_{x=1}^m q_{\alpha x}^{i-1} q_{x\beta}^{i-1} + q_{\alpha a}^{i-1} q_{b\beta}^{i-1} \tag{24}$$

Just like  $Q_{i-1}^2 = Q_{i-1}$  that

$$\sum_{x=1}^m q_{\alpha x}^{i-1} q_{x\beta}^{i-1} = q_{\alpha\beta}^{i-1} \tag{25}$$

Where do we get the formula for calculating the elements of the matrix:

$$q_{\alpha\beta}^i = q_{\alpha\beta}^{i-1} + q_{\alpha a}^{i-1} q_{b\beta}^{i-1} \tag{26}$$

*Remark 2.* The algorithm cannot be simplified by adding at once at each step several pairs satisfying the condition of the theorem. So in the example above:

$$(Q_1 \cup (2,4) \cup (4,1))^\tau = \begin{pmatrix} I & I & I & I & 0 \\ I & I & I & I & 0 \\ I & I & I & I & 0 \\ I & I & I & I & 0 \\ I & I & I & I & I \end{pmatrix}.$$

*Remark 3.* In the case where  $f(n) \leq \frac{n}{2}$ , the algorithm always finds a transitive modification, if it exists, since the group relation is linear. If  $f(n) > \frac{n}{2}$ , then, the algorithm, generally speaking, is not applicable. We give an appropriate example. Let the following individual relations be given:

- $R_1: 1 > 2 > 3 > 4 > 5,$
- $R_2: 2 = 3 > 4 > 1 = 5,$
- $R_3: 1 > 4 = 5 > 3 > 2,$
- $R_4: 1 = 2 = 3 = 5 > 4,$
- $R_5: 4 > 2 > 1 = 3 = 5,$
- $R_6: 5 > 1 \Rightarrow 4 > 3 > 2.$

Put  $f(n) = \frac{2}{3}$ . Then:

$$R = \begin{pmatrix} I & I & I & I & I \\ 0 & I & I & 0 & I \\ 0 & I & I & 0 & I \\ 0 & 0 & 0 & I & I \\ I & 0 & I & 0 & I \end{pmatrix}, P = P^\tau = \begin{pmatrix} I & I & I & I & I \\ 0 & I & 0 & 0 & I \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & I \\ 0 & 0 & 0 & 0 & I \end{pmatrix}.$$

$$(P \cup (5,3))^\tau = \begin{pmatrix} I & I & I & I & I \\ 0 & I & I & 0 & I \\ 0 & 0 & I & I & I \\ 0 & 0 & I & I & I \\ 0 & 0 & I & 0 & I \end{pmatrix} \notin R.$$

In this case, we can offer the following trick: we replace the relation  $R$  by the relation  $R_0 = R \cup (\overline{R} \cap \overline{R^{-1}})$ . It is easy to see that  $R_0$  is linear. Applying the algorithm to  $R_0$  now and putting  $Q_1 = P^\tau$ , we obtain a transitive relation, which will be quite close to the relation  $R$  (in the sense of the Hamming distance). So in the example above:

$$R_0 = \begin{pmatrix} I & I & I & I & I \\ 0 & I & I & I & I \\ 0 & I & I & I & I \\ 0 & I & I & I & I \\ I & 0 & I & 0 & I \end{pmatrix}, Q_1 = (P \cup (5, 3))^\tau = \begin{pmatrix} I & I & I & I & I \\ 0 & I & I & 0 & I \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & I & I & I \\ 0 & 0 & I & 0 & I \end{pmatrix}.$$

$$Q_2 = (Q_1 \cup (2,4))^\tau = \begin{pmatrix} I & I & I & I & I \\ 0 & I & I & I & I \\ 0 & 0 & I & 0 & I \\ 0 & 0 & I & I & I \\ 0 & 0 & I & 0 & I \end{pmatrix}.$$

The ratio  $Q_2$  is linear and therefore can be taken as the desired solution.

## 5. Discussion of the results of the algorithmic procedures for the harmonization of individual views in the implementation of organizational and managerial labor relations by the publisher

In the framework of this study, the algorithmization of the procedures for harmonizing individual views in the process of implementing organizational and managerial labor legal relations with a publishing house was developed. The proposed methodology is a continuation of research on the development of a mathematical apparatus for formalizing processes and research on the development of a methodology for labor efficiency, information support for publishing. Possible areas of practical application of the proposed methodology are:

- management of publishing projects;
- information support of publishing activities;
- legal regulation of labor at the enterprise.

Based on the results of the development, the following conclusions can be drawn regarding the possible practical application of the algorithmization of the procedures for harmonizing individual views in the process of implementing organizational and managerial labor relations by a publishing house.

1) A dialogue procedure has been developed for harmonizing individual views. At the same time, the aim was to ensure the «subtlety» of group choice. As the initial (basic) rules, rules of the majority type are used. Repeated appeal to individuals involves clarifying their opinions regarding alternatives that are part of the «best» group. Therefore, at each step of the procedure, the number of «best» alternatives obtained according to group selection does not increase. The process stops when one of the best alternatives out of the many compared remains, or when, after the next appeal to individuals, their opinions have not changed.

2) If the mathematical analysis is subjected to questions about which choice is «fair» or «reasonable», then the problem of group choice can be interpreted as follows. The situation is a common problem of a «reasonable» transition from given «individual data sets» to a single compact «group data set». As a result, the created algorithmization of the procedures for harmonizing individual views must be adapted to the specific conditions of the legal organization of labor, management of publishing projects.

3) In the process of implementing organizational and managerial labor relations by the publishing house, «individuals» can have a very different nature. For example, it can be experts, stakeholders, individual members of the publishing team, consumers.

4) Algorithmization of the procedures for the harmonization of individual opinions will improve local legal regulation at enterprises, modernize models for the implementation of organizational and managerial labor relations, adjust the labor function of publishing workers, and modernize the legal mechanism for protecting labor rights.

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